Barem de corectare OLM Clasa a VIII-a, 2014

1. a) $x^4+y^4+x^2+y^2+\sqrt{2}xy=\left(x^2+y^2\right)^2-2x^2y^2+x^2+y^2+\sqrt{2}xy=$ **...(1p)**

$=\left(x^2+y^2-\sqrt{2}xy\right)\left(x^2+y^2+\sqrt{2}xy\right)+\left(x^2+y^2+\sqrt{2}xy\right)=$ **...(1p)**

$=\left(x^2+y^2+\sqrt{2}xy\right)\left(x^2+y^2-\sqrt{2}xy+1\right)$ **...(1p)**

b) $x^2+y^2\geq 2xy=2$ **...(2p)**

$E(x,y)\geq\left(2+\sqrt{2}\right)\left(2-\sqrt{2}+1\right)$ **...(1p)**

$E(x,y)\geq 4+\sqrt{2}$ **...(1p)**

2. $\frac{1}{a+b}\leq\frac{a+b}{4ab}\Leftrightarrow(a-b)^2\geq 0$ **...(2p)**

$\frac{1}{a+b}\leq\frac{a+b}{4ab}=\frac{c(a+b)}{4abc}=\frac{ac+bc}{4}$ **...(2p)**

$\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}\leq\frac{2(ab+bc+ac)}{4}=\frac{ab+bc+ac}{2}$ **...(1p)**

$ab+ac+bc\leq a^2+b^2+c^2$ **...(1p)**

Finalizare **...(1p)**

3. a) ΔMNP echilateral de latură $\frac{a\sqrt{2}}{2}$ **...(1p)**

$PA=AN=AM=\frac{a}{2}$ **...(1p)**

Piramida $APMN$ triunghiulară regulată cu înălţimea $AO=h=\frac{a\sqrt{3}}{6}$ **...(1p)**

b) Se arată că planele $(MNP)\|(M'N'P')$ **...(1p)**

$AC'=d(A,(PMN))+d((MNP),(M'N'P'))+d(C'(M'N'P'))$ **...(1p)**

$AC'=a\sqrt{3}$ **...(1p)**

Distanţa dintre cele două plane este $\frac{2a\sqrt{3}}{3}$ **...(1p)**

4. a) $\Delta MNA(\prec N=90^0)\Rightarrow MA=4cm$ **...(1p)**

ΔMOA echilateral $\Rightarrow MO=4cm\Rightarrow A_{\Delta MBD}=12cm^2$ **...(2p)**

b) $MN\perp(AOD),\ NT\perp AD\Rightarrow MT\perp AD\Rightarrow d(M,AD)=MT$ **...(1p)**

OR înălţime în $\Delta AOD,\ OR=\frac{12}{5}cm$ **...(1p)**

NT linie mijlocie în $\Delta AOR\Rightarrow NT=\frac{6}{5}cm$ **...(1p)**

$\Delta MNT: MT=\frac{4\sqrt{21}}{5}cm$ **...(1p)**