

Concursul Interjudețean de Matematică „Cristian S. Calude”
ediția a XX-a
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Clasa a VII-a

BAREM DE CORECTARE NOTARE

Problema 1

$$\text{a) } \frac{2 \cdot a + 3 \cdot b}{4 \cdot a + 5 \cdot b + 6 \cdot c} = \frac{2 \cdot b + 3 \cdot c}{4 \cdot b + 5 \cdot c + 6 \cdot a} = \frac{2 \cdot c + 3 \cdot a}{4 \cdot c + 5 \cdot a + 6 \cdot b} = \frac{5 \cdot a + 5 \cdot b + 5 \cdot c}{15 \cdot a + 15 \cdot b + 15 \cdot c} = \frac{1}{3} \dots\dots\dots 1\text{p}$$

$$\sqrt{\frac{a+3 \cdot b}{3 \cdot a+4 \cdot b+5 \cdot c} + \frac{b+3 \cdot c}{3 \cdot b+4 \cdot c+5 \cdot a} + \frac{c+3 \cdot a}{3 \cdot c+4 \cdot a+5 \cdot b}} = \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1 \in \mathbb{N} \dots\dots\dots 2\text{p}$$

$$\text{b) } \frac{2 \cdot a + 3 \cdot b}{4 \cdot a + 5 \cdot b + 6 \cdot c} = \frac{1}{3} \Rightarrow 6 \cdot a + 9 \cdot b = 4 \cdot a + 5 \cdot b + 6 \cdot c \dots\dots\dots 1\text{p}$$

$$\Rightarrow a + 2 \cdot b = 3 \cdot c, b + 2 \cdot c = 3 \cdot a, c + 2 \cdot a = 3 \cdot b \Rightarrow a = b = c \dots\dots\dots 1\text{p}$$

$$\sqrt{\frac{\sqrt{5 \cdot a \cdot a}}{4 \cdot a + 5 \cdot a + 6 \cdot a}} \cdot 3 \leq \frac{\sqrt{2}}{2} \Leftrightarrow \sqrt{\frac{\sqrt{5 \cdot a}}{15 \cdot a}} \cdot 3 \leq \frac{\sqrt{2}}{2} \Leftrightarrow \sqrt{\frac{\sqrt{5}}{5}} \leq \frac{\sqrt{2}}{2} \dots\dots\dots 1\text{p}$$

$$\frac{5}{25} \leq \frac{4}{16} \Leftrightarrow \frac{1}{5} \leq \frac{1}{4} \Leftrightarrow 4 \leq 5 \dots\dots\dots 1\text{p}$$

Problema 2

a) PE este mediatoarea segm [AB] $\Rightarrow PE \perp BA$, D mijlocul lui [BC] $\Rightarrow AD \perp BC \dots\dots\dots 1\text{p}$
H este ortocentrul triunghiului ABP, deci BH \perp AP $\dots\dots\dots 1\text{p}$

b) ED \parallel AC $\Rightarrow \sphericalangle BED = \sphericalangle BAC = 30^\circ \dots\dots\dots 1\text{p}$

EF \parallel BC $\Rightarrow m(\sphericalangle AEF) = m(\sphericalangle ABC) = 75^\circ \Rightarrow m(\sphericalangle DEF) = 75^\circ \dots\dots\dots 1\text{p}$

EF \parallel BP $\Rightarrow m(\sphericalangle AFE) = m(\sphericalangle APB) = 30^\circ \dots\dots\dots 1\text{p}$

$\Rightarrow m(\sphericalangle PFD) = 120^\circ \Rightarrow m(\sphericalangle EFD) = 30^\circ \dots\dots\dots 1\text{p}$

$m(\sphericalangle DEF) = 75^\circ, m(\sphericalangle EFD) = 30^\circ, m(\sphericalangle EDF) = 75^\circ \dots\dots\dots 1\text{p}$

Problema 3

a) $m = 1008 \dots\dots\dots 1\text{p}$

Dacă $m \geq 1009$ proprietatea nu poate avea loc $\dots\dots\dots 1\text{p}$

Finalizare $\dots\dots\dots 1\text{p}$

b) $k = 0$, avem $a = \sqrt{5} \Rightarrow 2 < a < 3$, deci cei doi întregi sunt 2 și 3 $\dots\dots\dots 1\text{p}$

$k = 1 \Rightarrow a = \frac{\sqrt{2} + \sqrt{5}}{2}, k = 2 \Rightarrow a = \frac{2\sqrt{2} + \sqrt{5}}{3}$ obținem $1 < a < 2 \dots\dots\dots 1\text{p}$

Finalizare $\dots\dots\dots 2\text{p}$