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Societatea de Ştiinţe Matematice din România **Colegiul Naţional „Vasile Alecsandri”**

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Clasa a VIII-a

SOLUŢII

Problema 1.

Soluţie. a) $$\frac{1}{n\sqrt{n+1}+(n+1)\sqrt{n}}=\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}, \forall n\in\mathbb{N}^*$$

$$\frac{1}{n\sqrt{n+1}+(n+1)\sqrt{n}}=\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n(n+1)}}$$

$$\frac{1}{\sqrt{n(n+1)}\left(\sqrt{n}+\sqrt{n+1}\right)}=\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n(n+1)}}$$

$$\frac{1}{\sqrt{n(n+1)}\left(\sqrt{n}+\sqrt{n+1}\right)}=\frac{(n+1)-n}{\sqrt{n(n+1)}\left(\sqrt{n}+\sqrt{n+1}\right)}$$

b) $$\frac{1}{1\cdot\sqrt{2}+2\cdot\sqrt{1}}+\frac{1}{2\cdot\sqrt{3}+3\cdot\sqrt{2}}+...+\frac{1}{n\cdot\sqrt{n+1}+(n+1)\cdot\sqrt{n}}=1-\frac{\sqrt{2}}{4}$$

Aplicând punctul a) rezultă că: $$\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}=1-\frac{\sqrt{2}}{4}$$

$$1-\frac{1}{\sqrt{n+1}}=1-\frac{\sqrt{2}}{4}$$

$$\frac{1}{\sqrt{n+1}}=\frac{\sqrt{2}}{4}$$

$$n+1=8\Rightarrow n=7$$

Problema 2.

Soluţie. a) Fie $a,b,c\in\mathbb{R}$, $a^2+b^2+c^2=1$

$$(a+b+c)^2=a^2+b^2+c^2+2\cdot a\cdot b+2\cdot a\cdot c+2\cdot b\cdot c\geq 0$$

$$1+2\cdot(a\cdot b+a\cdot c+b\cdot c)\geq 0 \ \Rightarrow a\cdot b+a\cdot c+b\cdot c\geq -\frac{1}{2}$$

$$(a-b)^2+(a-c)^2+(b-c)^2\geq 0$$

$$a^2-2\cdot a\cdot b+b^2+a^2-2\cdot a\cdot c+c^2+b^2-2\cdot b\cdot c+c^2\geq 0$$

$2\cdot\left(a^2+b^2+c^2\right)\geq 2\cdot(a\cdot b+\cdot a\cdot c+b\cdot c)$

$a^2+b^2+c^2\geq a\cdot b+a\cdot c+b\cdot c$

$1\geq a\cdot b+a\cdot c+b\cdot c$

b) Fie $a,b,c\in\mathbb{R}_+^*$, $a\cdot b\cdot c\cdot d=1$

$a^2+b^2\geq 2\cdot a\cdot b$ şi $c^2+d^2\geq 2\cdot c\cdot d \Rightarrow a^2+b^2+c^2+d^2\geq 2\cdot(a\cdot b+c\cdot d)$

$a\cdot b+c\cdot d\geq 2\cdot\sqrt{a\cdot b\cdot c\cdot d}=2 \Rightarrow a^2+b^2+c^2+d^2\geq 2\cdot 2=4$

$a\cdot c+b\cdot d\geq 2\cdot\sqrt{a\cdot c\cdot b\cdot d}=2$

$a\cdot d+b\cdot c\geq 2\cdot\sqrt{a\cdot d\cdot b\cdot c}=2$

Rezultă că:

$a^2+b^2+c^2+d^2+a\cdot b+a\cdot c+a\cdot d+b\cdot c+b\cdot d+c\cdot d\geq 10$

Problema 3

Soluţie. a) $CM=\frac{1}{3}BC\Rightarrow\frac{CM}{BC}=\frac{1}{3}\Rightarrow\frac{CM}{BC-CM}=\frac{1}{3-1}\Rightarrow\frac{CM}{BM}=\frac{1}{2}$

Construim $CN\parallel AB, N\in AM$

$\Delta CMN\sim\Delta BMA\Rightarrow\frac{MN}{MA}=\frac{CN}{AB}=\frac{CM}{BM}=\frac{1}{2}$

$MN=\frac{1}{2}MA$ şi $CN=\frac{1}{2}AB$

În $\Delta ACN: AN<AC+CN\Rightarrow AM+MN<AC+CN$

$\Rightarrow AM+\frac{1}{2}AM<AC+\frac{1}{2}AB\Rightarrow\frac{3}{2}AM<AC+\frac{1}{2}AB\Rightarrow AM<\frac{2}{3}AC+\frac{1}{3}AB$

b) Fie I centrul cercului înscris ΔABC

$BI\cap DE=\{F\}$

$m(\sphericalangle IDA)=m(\sphericalangle IEA)=90^\circ$

$\Rightarrow ADIE$ patrulater inscriptibil

$\Rightarrow m(\sphericalangle DEI)=m(\sphericalangle DAI)$

$m(\sphericalangle DEI)=m(\sphericalangle IEA)-m(\sphericalangle AED)=90^\circ-m(\sphericalangle AED)$

$\sphericalangle AED\equiv\sphericalangle FEC$ (opuse la vârf)

$m(\sphericalangle DEI)=90^\circ-m(\sphericalangle FEC)\Rightarrow m(\sphericalangle DAI)=90^\circ-m(\sphericalangle FEC)\Rightarrow m(\sphericalangle FEC)=90^\circ-m(\sphericalangle DAI)$

$m(\sphericalangle FIC)=\frac{1}{2}\left(m(\sphericalangle ABC)+m(\sphericalangle ACB)\right)=90^\circ-\frac{1}{2}m(\sphericalangle BAC)=90^\circ-m(\sphericalangle DAI)$

Rezultă că: $\sphericalangle FIC\equiv\sphericalangle FEC \Rightarrow IEFC$ patrulater inscriptibil $\Rightarrow m(\sphericalangle IFC)=m(\sphericalangle IEC)=90^\circ$